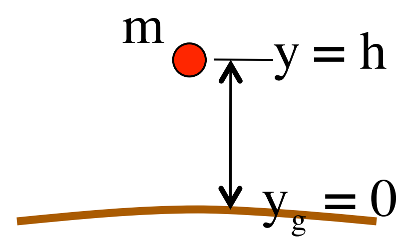
Problem 8.2



a.) Assuming the earth and mass are a part of one system (see sketch) and the two are near one another. For this case, the two-body system will have a gravitational energy function derived as "mgy," where "y" is a *coordinate* of the mass relative to the coordinate axis that has been set up. If we take the axis to be set up on the earth's surface, "y = h." (Note that things don't *have* to be set up that way. You get to choose where the zero level is ONLY because there is no preferred point where "F = 0" when you are dealing with gravity near the earth's surface. Anyway . . . writing the conservation of energy relationship in the form I generally use, we get:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 \quad + mg(y) + \quad 0 \quad = \frac{1}{2}mv_2^{\ 2} \ + \ 0$$

$$\Rightarrow \quad v_2 = (2gy)^{1/2}$$

$$= (2gh)^{1/2}$$

QUICK NOTE about the *MODIFIED CONSERVATION OF ENERGY* EXPRESSION:

As you know (or will soon learn) the *conservation of energy* basically is generated by using the Work/Energy Theorem, incorporating potential energy functions to determine work calculation, and rearranging. In its most succinct state, it maintains that the *total mechanical energy* at one point in time (E_1) will equal the *total mechanical energy* at a second point in time (E_2) unless some "extraneous work" (W_{ext}) is done that changes the energy content of the system. As *total mechanical energy* is defined as the sum of the kinetic and potential energy in the system $\left(\sum KE + \sum U\right)$ at a particular *point-in-time*, the conservation of energy can be written succinctly as:

$$E_1 + \sum W_{ext} = E_2$$

And in a more user-friendly form, it can be written as:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

This is the form I used in the previous section. I like it because it tells you EXACTLY WHAT TO DO in analyzing the problem. Specifically:

Is there:

--Any motion at *point-in-time 1*? If NO, $\sum KE_1$ is just zero (look at *Part a*—that is exactly what I put there). If there is, you write $\frac{1}{2}mv_1^2$ for each of the bodies that are moving, and you are done with that bailiwick.

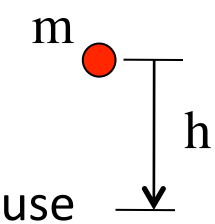
--Any potential energy in the system at *point-in-time* 1? If not, $\sum U_1$ is zero. If there is, you write mgy_1 if there is gravity and you are close to the earth's surface, or $\frac{1}{2}kx^2$ if there is a depressed spring, or whatever the potential energy function is for whatever force that is acting at that time.

--Any work being done between *point-in-time* 1 and *point-in-time* 2 that is NOT being taken care of with a potential energy function? If not, there is no "extraneous work" being done and the $\sum W_{ext}$ bailiwick will be zero. If so (if there is, for instance, a non-conservative force like friction in the system), you have to calculate $W = \vec{F} \bullet \vec{d}$ or $W = \int \vec{F} \bullet d\vec{r}$, depending, for the force or forces in question, and place those results in the $\sum W_{ext}$ bailiwick.

--Any motion at *point-in-time 2*? If not, $\sum KE_2$ is just zero. Otherwise, write out $\frac{1}{2}mv_2^2$ for each moving body.

--Any potential energy in the system at *point-in-time* 2? If not, $\sum U_2$ is zero. Otherwise, etc.

b.) Assuming the mass is an entity by itself, what would the *modified conservation of energy* expression look like in that case?



In this case, because the earth is not a part of the system, we can't use the potential energy function as we did before when "the system" was a two-body set-up. Now, instead, we have to treat the work the earth does on the mass as an "extraneous" amount of work, and use $W = \vec{F} \bullet \vec{d}$ to determine that value. Noting that the angle between the displacement vector and the gravitation vector is *zero* (they are both downward), we can write:

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

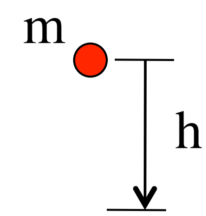
$$0 \quad + \quad 0 \quad + \quad \vec{F} \bullet \vec{d} \quad = \quad \frac{1}{2} m v_2{}^2 \quad + \quad 0$$

$$\Rightarrow \qquad \left|\vec{F}\right|\left|\vec{d}\right| \cos 0^{o} = \frac{1}{2} m v_2{}^2$$

$$\Rightarrow \qquad (mg)(h)(1) = \frac{1}{2} m v_2{}^2$$

$$\Rightarrow \qquad v = (2gh)^{1/2}$$

c.) How do the two velocities compare?



As expected, they are the same. What's important to note from this exercise is that:

--If work is done by a force field generated by something that is a part of the system, and if you know the potential energy function for that force field, you take the work that force does into account in the *summation of potential energies* part of the *conservation of energy relationship.*

--If work is done by a force field generated by something NOT a part of the system, the work that force field does is listed in the *extraneous work* bailiwick. Indeed, you might determine that work quantity using $W = \vec{F} \bullet \vec{d}$, or you might be able to determine it with a potential energy function (which you could have done here with $W = -\left(mgy_{final} - mgy_{initial}\right)$), but if the force source is defined as outside the system, the actual work quantity will be placed in the *conservation of energy relationship* in the *extraneous work* pot, not either of the *potential energy* pots.